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An Advanced Stochastic Model for Threshold Crossing Studies of Rotor Blade Vibrations

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A STOCHASTIC model to analyze turbulence excited rotor blade vibrations, previously described by the authors, has been generalized and amplified to include nonuniformity of the atmospheric turbulence velocity across the rotor disk in longitudinal direction.

The authors[1] have previously solved the problem of threshold crossing expectations of rotor blade flapping response to atmospheric turbulence. The theory pertained to high rotor advance ratios and low-rotor lift, where a linear description is adequate and where turbulence excited blade vibrations are severe. In the earlier stochastic model it has been assumed that the vertical turbulence velocity at a given point in time is uniform over the rotor disc. This assumption limited the theory to turbulence scale lengths which are large as compared to the rotor radius. In the following a less restrictive stochastic model is described where correlations between vertical turbulence velocities across the rotor disk in the longitudinal direction are considered.

As before, the widely used von Kármán vertical turbulence spectrum is approximated by one with an exponential autocorrelation function and zero mean

$$R_\lambda(\tau)/\sigma_\lambda{}^2 = \exp - |r|\, 2/L \tag{1}$$

The associated two-sided spatial power spectral density is

$$\{S_\lambda(\omega_r)/\sigma_\lambda{}^2\}\, d\omega_r = \{2L/\pi[4+(\omega_r L)^2]\}\, d\omega_r \tag{2}$$

$\lambda = w/\Omega R$ is the dimensionless vertical turbulence velocity, Ω is the angular rotor speed, R the rotor radius, L is the longitudinal turbulence scale, $L/2$ the vertical turbulence scale. The integral over Eq. (2) from $-\infty$ to $+\infty$ is one. If r is the longitudinal displacement of the rotor center and if the flight velocity V is uniform, we have $r = Vt/\Omega$, where t is the dimensionless time with $1/\Omega$ as time unit. Defining the timewise dimensionless circular frequency ω associated with the spatial circular frequency ω_r by $\omega\Omega = \omega_r V$, one obtains from Eq. (2) the two-sided timewise power spectral density for the nondimensional vertical turbulence velocity λ

$$\{S_\lambda(\omega)/\sigma_\lambda{}^2\}\, d\omega = \{a/\pi(a^2+\omega^2)\}\, d\omega \tag{3}$$

where

$$a = 2V/\Omega L = 2\mu/(L/R) \tag{4}$$

$\mu = V/\Omega R$ is the rotor advance ratio. The spectrum Eq. (3) can be obtained by passing white noise through the first order filter

$$\dot\lambda + a\lambda = \sigma_\lambda(2a)^{1/2} n, \quad \text{with} \quad R_n(\tau) = \delta(\tau) \tag{5}$$

The integral over Eq. (3) from $-\infty$ to $+\infty$ is again one. Assuming that the dynamics of the rotor is given in state variable form by

$$\dot X(t) = A(t)X(t) + B(t)\lambda(t) \tag{6}$$

where $X(t)$ is the state vector with components $X_1, X_2, \ldots$ and with initial state $X(0)$, where $A(t)$ is the periodic state matrix, where $B(t)$ is the periodic coupling matrix relating the input vector $\lambda(t)$ assumed to have zero mean to the rate of state vector, one can express the response covariance matrix by

$$R_{xx}(t_1, t_2) = \int_{-\infty}^{\infty} H^*(\omega, t_1) S_\lambda(\omega) H^T(\omega, t_2)\, d\omega \tag{7}$$

$H(\omega, t)$ is the response vector to the input $\lambda(t) = u(t)\exp i\omega t$, $u(t)$ being the unit step function. Equations (6) and (7) are matrix generalizations to the expressions given earlier.[1] For the blade flapping problem $X_1 = \beta$, $X_2 = \dot\beta$, $\dot X_1 = \dot\beta$, $\dot X_2 = \ddot\beta$ etc. Once the response covariance matrix is known, and assuming that the input $\lambda(t)$ is Gaussian, it is easy to compute the threshold crossing expectations for any response quantity.[1,2] If $X_i = \dot X_k$, then the expected number of positive crossings of the level ζ per unit time is‡

$$\begin{aligned} E_k[N_+(\zeta, t)] = {} & (1/2\pi)(1-r_{ik}{}^2)^{1/2}(\sigma_i/\sigma_k) \times \\ & \exp[-(\zeta/\sigma_k)^2/2(1-r_{ik}{}^2)] + \\ & [1/2(2\pi)^{1/2}](\sigma_i/\sigma_k)(\zeta r_{ik}/\sigma_k)\{\exp[-(\zeta^2/2\sigma_k{}^2)]\} \times \\ & (1+\operatorname{erf}\{\zeta r_{ik}/\sigma_k[2(1-r_{ik}{}^2)^{1/2}]\}) \end{aligned} \tag{8}$$

If the filter Eq. (5) is included in the dynamic matrix Eq. (6) one has the case of white noise input, for which the response variance $R_{xx}(t,t) = P(t)$ can be determined from[3-5]

$$\dot P(t) = A(t)P(t) + P(t)A^T(t) + B(t)B^T(t) \tag{9}$$

with zero initial state. The response covariance matrix is then obtained from

$$R_{xx}(t_1, t_2) = \begin{cases} \Phi(t_1, t_2)P(t_2), & t_1 \geqslant t_2 \\ P(t_1)\Phi^T(t_2, t_1), & t_1 \leqq t_2 \end{cases} \tag{10}$$

where $\Phi(t, \tau)$ is the state transition matrix defined by $\dot\Phi(t, \tau) = A(t)\Phi(t, \tau)$, $\Phi(\tau, \tau) = I$. Under certain conditions the evaluation of Eqs. (9) and (10) requires less computational effort than that of Eq. (7).

So far we have merely generalized the previous stochastic model[1] to arbitrary many degrees of freedom. Now let us assume that the blade, instead of being subjected at each point in time to the vertical turbulence velocity λ at the rotor center, is subjected to the turbulence velocity at the 0.7 radius station, whereby this velocity is assumed to be uniform in the lateral direction, an assumption often made in aircraft turbulence response analysis. We now determine the response of the rotor blade as it passes through a space wave,

$$\lambda(r) = u(r)\exp i\omega_r r \tag{11}$$

With the previous assumptions $r = Vt/\Omega$ and $\omega\Omega = \omega_r V$ this is

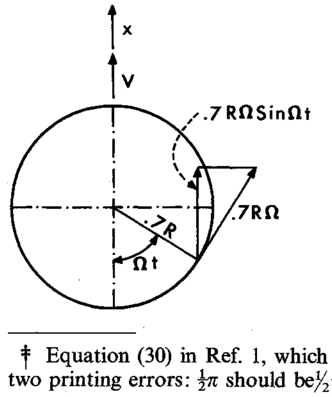

Fig. 1 Forward component of blade velocity at 0.7R.

Received January 14, 1972. Sponsored by the Ames Directorate, AAMRDL under Contract NAS2-4151.

Index categories: VTOL Structural Design (Including Loads); VTOL Vibration; Structural Dynamic Analysis.

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‡ Equation (30) in Ref. 1, which corresponds to Eq. (8) herein has two printing errors: $\frac{1}{2}\pi$ should be $1/2\pi$, $\frac{1}{2}(2\pi)^{1/2}$ should be $1/2(2\pi)^{1/2}$.

identical to the timewise input used for $H(\omega, t)$ in Eq. (7). Now, however, the relation between the longitudinal distance r traveled by the $0.7R$ station of the blade and the nondimensional time t must be derived. As seen from Fig. 1 the forward component of blade velocity at the $0.7R$ station is with the nondimensional time t (time unit $1/\Omega$)

$$dx/dt = V/\Omega + 0.7R \sin t \tag{12}$$

or integrated

$$x = Vt/\Omega - 0.7R \cos t + c \tag{13}$$

Setting $r = x_2 - x_1$ and assuming $t = 0$ for $x = x_1$ one obtains

$$r = Vt/\Omega - 0.7R \cos t \tag{14}$$

which inserted into Eq. (11) yields with $\omega\Omega = \omega_r V$ the timewise input

$$\lambda(t) = u(t) \exp i\omega[t - (0.7/\mu)\cos t] \tag{15}$$

If we interpret now $H(\omega, t)$ as the response to the delayed time input Eq. (15), the response covariance matrix is still given by Eq. (7). Numerically, using for example a Runge-Kutta integration routine for computing $H(\omega, t)$, the input Eq. (15) is as easily handled as the input without the time delay term $(0.7/\mu)\cos t$. However the filtered white noise method of Eqs. (9) and (10) is now not applicable. The problem can also be solved with the help of the nonstationary autocorrelation function. From Eq. (13) we have

$$r = x_2 - x_1 = (V/\Omega)(t_2 - t_1) - 0.7R(\cos t_2 - \cos t_1) \tag{16}$$

Inserting Eq. (16) into Eq. (1) one obtains

$$R_\lambda(t_1, t_2)/\sigma_\lambda{}^2 = \exp - |a(t_2 - t_1) - 1.4(R/L)(\cos t_2 - \cos t_1)| \tag{17}$$

The authors[6] have used a weighting function method to obtain the response covariance to the nonstationary random input process with the autocorrelation function Eq. (17). It was found that this method gave the same result as Eq. (7) with $H(\omega, t)$ from Eq. (15), however with a greater computational effort.

A numerical example used earlier[1] with rotor advance ratio $\mu = 1.6$, Lock number $\gamma = 4$, blade tip loss factor $B = 0.97$, blade flapping frequency ratio $P = 1.3$ was studied for a ratio of turbulence scale over rotor radius of $L/R = 4.0$, using Eq. (15) both with and without the time delay term $(0.7\mu)/\cos t$. It was found that the time variable flapping variance and the threshold crossing expectations for flapping in response to atmospheric turbulence were not substantially affected by the time delay term in Eq. (15). A typical turbulence scale at low altitude where turbulence is most severe is about $L = 400$ ft (Ref. 7). A value of $L/R = 4$ corresponds then to a rotor diameter of 100 ft which is much larger than for current or even for foreseeable future lifting rotors. It would then appear that the nonuniformity of the vertical turbulence velocity over the rotor disk is of little influence on the random blade flapping response, at least as far as longitudinal nonuniformity is concerned. This may not be true for more accurate blade representations including for example additional torsional and bending degrees of freedom, for which the preceding analysis remains valid. Also, lateral nonuniformity of the vertical turbulence velocity, because it is more elaborate to evaluate, has not been considered as yet.

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Panel-Flutter Analysis of a Thermal Protection-Shield Concept for the Space Shuttle

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Nomenclature

$A_{ij}{}^*$ = dimensionless generalized-aerodynamic-force matrix elements
B = number of panel segments
g = coefficient of nonviscous structural damping
$h_j(x)$ = shape of displacement mode j, length
I_s = mass moment of inertia of panel segment about its mass centroid, mass $(\text{length})^2$/unit width
$\tilde{k}$ = spring stiffness at supports, force/unit displacement per unit width
k_s = reduced frequency based on l_s, $\omega l_s/V$
l_s = length of panel segment
M = Mach number of freestream
$M_{ij}{}^*$ = dimensionless generalized-mass matrix element
m_s = mass of panel segment per unit width
Q_j = the mode-j aerodynamic-force term in the virtual-work series, force/unit width
$\bar{q}_j$ = generalized coordinate of displacement mode j
q_j = complex amplitude of $\bar{q}_j$
r_s = radius of gyration of panel segment about its mass centroid, length
U = elastic potential energy of the panel springs, force length/unit width
V = kinetic energy of the panel, force length/unit width
V = velocity of freestream, length/time
w = downwash at pnel surface, positive with z, length/time
w_j = the mode-j term in the series for w
x = streamwise coordinate, length
x_j = x-coordinate of segment juncture j ($j = 0, 1, \ldots, B$), length
z = vertical coordinate and vertical displacement of panel
z_j = the mode-j term in the series for z
Δp = lifting pressure, positive with z, force/unit area
δW = virtual work of the panel, force length/unit width
λ = dynamic pressure parameter $\rho V^2/M\tilde{k}$
μ = ratio of mass of panel segment to mass of air contained to a height of l_s over the segment, $m_s/\rho l_s{}^2$
ρ = density of airstream, mass/length^3
τ = time
Ω = flexibility eigenvalue, $(\omega_0/\omega)^2(1+ig)$
ω = circular frequency, time^{-1}
ω_0 = reference frequency, that of a mass m_s on a spring $\tilde{k}$, $(\tilde{k}/m_s)^{1/2}$, time^{-1}

Introduction

THE thermal protection shield of the space shuttle, like other structural elements, should be as low in weight as possible. This requirement leads to a need to insure the flutter safety of a shield during both launch and re-entry. The present Note describes a preliminary analysis based on a linear force-displacement relationship and piston-theory aerodynamic forces.

Received January 17, 1972; revision received February 25, 1972.
Index categories: Launch Vehicle and Missile Structural Design; Nonsteady Aerodynamics; Aeroelasticity and Hydroelasticity.

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